

List of Examples and Statements

1.1	Example: Sum of quadratic functions	13
1.2	Lemma: Hautus Lemma for controllability	23
1.3	Lemma: LQR convergence	23
1.4	Lemma: Hautus Lemma for observability	41
1.5	Lemma: Convergence of estimator cost	42
1.6	Lemma: Estimator convergence	42
1.7	Assumption: Target feasibility and uniqueness	46
1.8	Lemma: Detectability of the augmented system	49
1.9	Corollary: Dimension of the disturbance	49
1.10	Lemma: Offset-free control	51
1.11	Example: More measured outputs than inputs and zero offset	53
1.12	Lemma: Hautus Lemma for stabilizability	68
1.13	Lemma: Hautus Lemma for detectability	72
1.14	Lemma: Stabilizable systems and feasible targets	83
2.1	Proposition: Continuous system solution	94
2.2	Assumption: Continuity of system and cost	97
2.3	Assumption: Properties of constraint sets	97
2.4	Proposition: Existence of solution to optimal control problem	97
2.5	Example: Linear quadratic MPC	99
2.6	Example: Closer inspection of linear quadratic MPC	101
2.7	Theorem: Continuity of value function and control law	104
2.8	Example: Discontinuous MPC control law	104
2.9	Definition: Feasible preimage of the state	108
2.10	Definition: Positive and control invariant sets	110
2.11	Proposition: Existence of solutions to DP recursion	110
2.12	Assumption: Basic stability assumption	115
2.13	Assumption: Implied invariance assumption	115
2.14	Lemma: Optimal cost decrease	115
2.15	Lemma: Monotonicity of the value function	116
2.16	Assumption: Bounds on stage and terminal costs	118
2.17	Proposition: Optimal value function properties	118
2.18	Proposition: Extension of upper bound to compact set	119
2.19	Proposition: Lyapunov function on \mathcal{X}_N	120

2.20	Proposition: Boundedness of X_j	120
2.21	Proposition: Properties of discrete time system	121
2.22	Theorem: Asymptotic stability with unbounded region of attraction	122
2.23	Assumption: Weak controllability	122
2.24	Theorem: MPC stability	123
2.25	Assumption: Continuity of system and cost; time-varying case	127
2.26	Assumption: Properties of constraint sets; time-varying case	128
2.27	Definition: Time-varying control invariant sets	128
2.28	Proposition: Continuous system solution; time-varying case	128
2.29	Proposition: Existence of solution to optimal control problem; time-varying case	128
2.30	Assumption: Basic stability assumption; time-varying case	129
2.31	Assumption: Implied invariance assumption; time-varying case	129
2.32	Lemma: Optimal cost decrease; time-varying case	129
2.33	Lemma: MPC cost is less than terminal cost	129
2.34	Assumption: Bounds on stage and terminal costs; time-varying case	130
2.35	Proposition: Optimal value function properties; time-varying case	130
2.36	Assumption: Uniform weak controllability	130
2.37	Theorem: MPC stability; time-varying case	131
2.38	Lemma: Entering the terminal region	150
2.39	Theorem: MPC stability; no terminal constraint	150
2.40	Definition: Input/output-to-state stable (IOSS)	153
2.41	Proposition: Convergence of state under IOSS	154
2.42	Theorem: MPC stability with unreachable setpoint	164
2.43	Example: Unreachable setpoint MPC	164
2.44	Theorem: Lyapunov theorem for asymptotic stability	177
2.45	Lemma: An equality for quadratic functions	178
2.46	Lemma: Evolution in a compact set	179
3.1	Definition: Robust global asymptotic stability (GAS)	201
3.2	Theorem: Lyapunov function and robust GAS	201
3.3	Theorem: Robust GAS and regularization	202
3.4	Assumption: Lipschitz continuity of value function	204
3.5	Assumption: Restricted disturbances	206
3.6	Definition: Robust control invariance	210

3.7	Definition: Robust positive invariance	210
3.8	Assumption: Basic stability assumption; robust case	210
3.9	Assumption: Implied stability assumption; robust case	210
3.10	Theorem: Recursive feasibility of control policies	212
3.11	Assumption: Existence of robust control invariant set	217
3.12	Assumption: Properties of robust control invariant set	217
3.13	Definition: Set algebra and Hausdorff distance	220
3.14	Assumption: Restricted disturbances for constraint satisfaction	227
3.15	Proposition: Exponential stability of tube-based MPC	230
3.16	Assumption: Compact convex disturbance set	232
3.17	Proposition: Exponential stability of tube-based MPC without nominal trajectory	237
3.18	Assumption: Quadratic stabilizability	238
3.19	Assumption: Restricted parameter uncertainty	239
3.20	Proposition: Asymptotic stability of tube-based MPC	241
3.21	Proposition: Existence of tubes for nonlinear systems	249
3.22	Example: Robust control of an exothermic reaction	251
4.1	Definition: i-IOSS	267
4.2	Proposition: Convergence of state under i-IOSS	268
4.3	Assumption: Convergent disturbances	268
4.4	Remark: Summable disturbances	268
4.5	Assumption: Positive definite stage cost	269
4.6	Definition: Global asymptotic stability	269
4.7	Definition: Robust global asymptotic stability	270
4.8	Theorem: Robust GAS of full information estimates	270
4.9	Lemma: Duality of controllability and observability	276
4.10	Theorem: Riccati iteration and regulator stability	276
4.11	Definition: Observability	278
4.12	Definition: Final-state observability	278
4.13	Theorem: Robust GAS of MHE with zero prior weighting	279
4.14	Definition: Full information arrival cost	281
4.15	Lemma: MHE and full information estimation	281
4.16	Definition: MHE arrival cost	281
4.17	Assumption: Prior weighting	282
4.18	Proposition: Arrival cost of full information greater than MHE	282
4.19	Assumption: MHE detectable system	283
4.20	Theorem: Robust GAS of MHE	283

4.21	Assumption: Estimator constraint sets	287
4.22	Theorem: Robust GAS of constrained full information . . .	288
4.23	Theorem: Robust GAS of constrained MHE	289
4.24	Assumption: Prior weighting for linear system	289
4.25	Assumption: Polyhedral constraint sets	290
4.26	Corollary: Robust GAS of constrained MHE	290
4.27	Example: EKF and UKF	296
4.28	Example: Sampled density of the lognormal	305
4.29	Theorem: Kolmogoroff (1933)	310
4.30	Example: Sampling error distribution for many samples . .	312
4.31	Example: Sampling independent random variables	313
4.32	Example: Sampling a conditional density	315
4.33	Example: Importance sampling of a multimodal density . .	319
4.34	Example: Importance sampling of a multimodal function .	323
4.35	Theorem: Resampling	326
4.36	Example: Resampling a bimodal density	328
4.37	Example: What's wrong with the simplest particle filter? .	335
4.38	Example: Can resampling fix the simplest particle filter? .	337
4.39	Example: Optimal importance function applied to a linear estimation problem	345
4.40	Example: Comparison of MHE, PF, and combined MHE/PF .	351
4.41	Theorem: Resampling and pruning	362
5.1	Definition: Positive invariance; robust positive invariance .	377
5.2	Proposition: Proximity of state and state estimate	378
5.3	Proposition: Proximity of state estimate and nominal state	380
5.4	Assumption: Constraint bounds	381
5.5	Proposition: Exponential stability of output MPC	383
5.6	Definition: Positive invariance; time-varying case	385
5.7	Definition: Robust positive invariance; time-varying case .	385
5.8	Proposition: Properties of composite system	387
5.9	Assumption: Constraint bounds; time-varying case	388
5.10	Proposition: Exponential convergence of output MPC: time- varying case	390
6.1	Definition: Lyapunov stability	417
6.2	Definition: Uniform Lyapunov stability	417
6.3	Definition: Exponential stability	418
6.4	Lemma: Exponential stability of suboptimal MPC	418
6.5	Lemma: Exponential stability with mixed powers of norm .	420

6.6	Lemma: Converse theorem for exponential stability	420
6.7	Assumption: Unconstrained two-player game	427
6.8	Example: Nash equilibrium is unstable	430
6.9	Example: Nash equilibrium is stable but closed loop is unstable	431
6.10	Example: Nash equilibrium is stable and the closed loop is stable	432
6.11	Example: Stability and offset in the distributed target cal- culation	441
6.12	Assumption: Constrained two-player game	447
6.13	Lemma: Exponential stability of perturbed system	454
6.14	Assumption: Disturbance models	455
6.15	Lemma: Detectability of distributed disturbance model	455
6.16	Assumption: Constrained M -player game	460
6.12	Assumption: Constrained two-player game	475
6.17	Lemma: Local detectability	475
7.1	Definition: Polytopic (polyhedral) partition	488
7.2	Definition: Piecewise affine function	488
7.3	Assumption: Strict convexity	489
7.4	Definition: Polar cone	491
7.5	Proposition: Farkas's Lemma	491
7.6	Proposition: Optimality conditions for convex set	491
7.7	Proposition: Optimality conditions in terms of polar cone	493
7.8	Proposition: Optimality conditions for linear inequalities	493
7.9	Proposition: Solution of $\mathbb{P}(w)$, $w \in R_x^0$	495
7.10	Proposition: Piecewise quadratic (affine) cost (solution)	496
7.11	Example: Parametric QP	496
7.12	Example: Explicit optimal control	497
7.13	Proposition: Continuity of cost and solution	499
7.14	Assumption: Continuous, piecewise quadratic function	502
7.15	Definition: Active polytope (polyhedron)	503
7.16	Proposition: Solving \mathbb{P} using \mathbb{P}_i	503
7.17	Proposition: Optimality of $u_x^0(w)$ in R_x	506
7.18	Proposition: Piecewise quadratic (affine) solution	506
7.19	Proposition: Optimality conditions for parametric linear program	510
7.20	Proposition: Solution of \mathbb{P}	513
7.21	Proposition: Piecewise affine cost and solution	513

A.1	Theorem: Schur decomposition	539
A.2	Theorem: Real Schur decomposition	540
A.3	Theorem: Bolzano-Weierstrass	542
A.4	Proposition: Convergence of monotone sequences	542
A.5	Proposition: Uniform continuity	544
A.6	Proposition: Compactness of continuous functions of compact sets	544
A.7	Proposition: Weierstrass	545
A.8	Proposition: Derivative and partial derivative	547
A.9	Proposition: Continuous partial derivatives	548
A.10	Proposition: Chain rule	548
A.11	Proposition: Mean value theorem for vector functions	548
A.12	Definition: Convex set	551
A.13	Theorem: Caratheodory	551
A.14	Theorem: Separation of convex sets	552
A.15	Theorem: Separation of convex set from zero	553
A.16	Corollary: Existence of separating hyperplane	553
A.17	Definition: Support hyperplane	554
A.18	Theorem: Convex set and halfspaces	554
A.19	Definition: Convex cone	554
A.20	Definition: Polar cone	554
A.21	Definition: Cone generator	555
A.22	Proposition: Cone and polar cone generator	555
A.23	Theorem: Convexity implies continuity	557
A.24	Theorem: Differentiability and convexity	557
A.25	Theorem: Second derivative and convexity	557
A.26	Definition: Level set	558
A.27	Definition: Sublevel set	558
A.28	Definition: Support function	558
A.29	Proposition: Set membership and support function	558
A.30	Proposition: Lipschitz continuity of support function	558
A.31	Theorem: Existence of solution to differential equations	561
A.32	Theorem: Maximal interval of existence	561
A.33	Theorem: Continuity of solution to differential equation	561
A.34	Theorem: Bellman-Gronwall	561
A.35	Theorem: Existence of solutions to forced systems	563
A.36	Example: Fourier transform of the normal density.	568
A.37	Example: Transform of the multivariate normal density	571
A.38	Example: Marginal normal density	573
A.39	Example: Nonlinear transformation	576

A.40	Example: Maximum of two random variables	577
A.41	Example: Independent implies uncorrelated	578
A.42	Example: Does uncorrelated imply independent?	579
A.43	Example: Independent and uncorrelated are equivalent for normals	581
A.44	Example: Conditional normal density	584
A.45	Example: More normal conditional densities	585
B.1	Definition: Equilibrium point	604
B.2	Definition: Positive invariant set	604
B.3	Definition: \mathcal{K} function	605
B.4	Definition: Local stability	605
B.5	Definition: Global attraction	605
B.6	Definition: Global asymptotic stability (GAS)	605
B.7	Definition: Various forms of stability	606
B.8	Proposition: GAS and comparison function	607
B.9	Definition: GAS for constrained systems	608
B.10	Definition: Lyapunov function	609
B.11	Theorem: Lyapunov function and GAS	609
B.12	Theorem: Converse theorem for asymptotic stability	610
B.13	Theorem: Lyapunov function for asymptotic stability (con- strained case)	610
B.14	Theorem: Lyapunov function for exponential stability	610
B.15	Lemma: Lyapunov function for linear systems	611
B.16	Definition: Nominal robust global asymptotic stability	612
B.17	Theorem: Nominal RGAS and Lyapunov function	612
B.18	Definition: Positive invariance with disturbances	613
B.19	Definition: Local stability (disturbances)	614
B.20	Definition: Global attraction (disturbances)	614
B.21	Definition: GAS (disturbances)	614
B.22	Definition: Lyapunov function (disturbances)	614
B.23	Theorem: Lyapunov function for GAS (disturbances)	614
B.24	Definition: Global control-Lyapunov function (CLF)	615
B.25	Definition: Global stabilizability	615
B.26	Definition: Positive invariance (disturbance and control)	616
B.27	Definition: CLF (disturbance and control)	616
B.28	Remark: CLF implies control law	616
B.29	Definition: Positive invariance (constrained)	617
B.30	Definition: CLF (constrained)	617
B.31	Definition: Control invariance (disturbances, constrained)	617

B.32	Definition: CLF (disturbances, constrained)	617
B.33	Definition: Input-to-state stable (ISS)	618
B.34	Definition: ISS-Lyapunov function	619
B.35	Lemma: ISS-Lyapunov function implies ISS	619
B.36	Definition: ISS (constrained)	619
B.37	Definition: ISS-Lyapunov function (constrained)	619
B.38	Lemma: ISS-Lyapunov function implies ISS (constrained)	620
B.39	Definition: Output-to-state stable (OSS)	620
B.40	Definition: OSS-Lyapunov function	620
B.41	Theorem: OSS and OSS-Lyapunov function	621
B.42	Definition: Input/output-to-state stable (IOSS)	621
B.43	Definition: IOSS-Lyapunov function	622
B.44	Conjecture: IOSS and IOSS-Lyapunov function	622
B.45	Definition: Incremental input/output-to-state stable	622
B.46	Definition: Observability	622
B.47	Assumption: Lipschitz continuity of model	623
B.48	Lemma: Lipschitz continuity and state difference bound	623
B.49	Theorem: Observability and convergence of state	623
B.50	Proposition: Improving convergence (Sontag (1998b))	625
C.1	Lemma: Principle of optimality	632
C.2	Theorem: Optimal value function and control law from DP	632
C.3	Example: DP applied to linear quadratic regulator	634
C.4	Definition: Tangent vector	637
C.5	Proposition: Tangent vectors are closed cone	637
C.6	Definition: Regular normal	637
C.7	Proposition: Relation of normal and tangent cones	638
C.8	Proposition: Global optimality for convex problems	639
C.9	Proposition: Optimality conditions – normal cone	640
C.10	Proposition: Optimality conditions – tangent cone	640
C.11	Proposition: Representation of tangent and normal cones	641
C.12	Proposition: Optimality conditions — linear inequalities	642
C.13	Corollary: Optimality conditions — linear inequalities	642
C.14	Proposition: Necessary condition for nonconvex problem	643
C.15	Definition: General normal	646
C.16	Definition: General tangent	646
C.17	Proposition: Set of regular tangents is closed convex cone	646
C.18	Definition: Regular set	647
C.19	Proposition: Conditions for regular set	647
C.20	Proposition: Quasiregular set	648

C.21 Proposition: Optimality conditions nonconvex problem . .	650
C.22 Proposition: Fritz-John necessary conditions	651
C.23 Definition: Outer semicontinuous function	655
C.24 Definition: Inner semicontinuous function	656
C.25 Definition: Continuous function	656
C.26 Theorem: Equivalent conditions for outer and inner semi- continuity	657
C.27 Proposition: Outer semicontinuity and closed graph	657
C.28 Theorem: Minimum theorem	658
C.29 Theorem: Lipschitz continuity of value function	659
C.30 Definition: Subgradient of convex function	660
C.31 Theorem: Clarke et al. (1998)	660
C.32 Corollary: Distance from a polyhedron	661
C.33 Theorem: Continuity of a polyhedral projection	663
C.34 Theorem: Continuity of the value function	663
C.35 Theorem: Lipschitz continuity of the value function	664