

List of Examples and Statements

1.1	Definition: Linear space	1
1.2	Definition: Subspace	2
1.3	Definition: Norm	2
1.4	Example: Common transformations do not commute . . .	7
1.5	Example: Matrix identities derived with index notation . .	10
1.6	Theorem: QR decomposition	11
1.7	Theorem: Existence and uniqueness of solutions for square systems	15
1.8	Example: Linearly independent columns, rows of a matrix	19
1.9	Definition: Rank of a matrix	20
1.10	Example: The geometry of least squares	24
1.11	Theorem: Self-adjoint matrix decomposition	37
1.12	Example: A nonsymmetric matrix	39
1.13	Example: A defective matrix	40
1.14	Example: Vibrational modes of a molecule	43
1.15	Theorem: Schur decomposition	46
1.16	Theorem: Symmetric Schur decomposition	47
1.17	Theorem: Real Schur decomposition	48
1.18	Definition: Convex function	58
1.19	Proposition: Full rank of $A^T A$	82
2.1	Example: Particle motion	106
2.2	Example: A forced first-order differential equation	109
2.3	Example: Sets of coupled first-order differential equations	109
2.4	Example: Power series solution for a constant-coefficient equation	115
2.5	Example: Frobenius solution for Bessel's equation of order zero	117
2.6	Example: Fourier series of a nonperiodic function	124
2.7	Example: Generating trigonometric basis functions	130
2.8	Example: Bessel's equation revisited	131
2.9	Example: Legendre's differential equation and Legendre polynomials	132
2.10	Theorem: Alternative theorem	133

2.11	Example: Steady-state temperature profile with fixed end temperatures	134
2.12	Example: Steady-state temperature profile with insulated ends	135
2.13	Example: Steady-state temperature profile with fixed flux	137
2.14	Example: Fixed flux revisited	141
2.15	Example: Nonhomogeneous boundary-value problem and the Green's function	142
2.16	Definition: (Lyapunov) Stability	147
2.17	Definition: Attractivity	147
2.18	Definition: Asymptotic stability	147
2.19	Definition: Exponential stability	148
2.20	Definition: Lyapunov function	149
2.21	Theorem: Lyapunov stability	150
2.22	Theorem: Asymptotic stability	151
2.23	Theorem: Exponential stability	152
2.24	Theorem: Lyapunov function for linear systems	155
2.25	Definition: Exponential stability (discrete time)	156
2.26	Definition: Lyapunov function (discrete time)	156
2.27	Theorem: Lyapunov stability (discrete time)	157
2.28	Theorem: Asymptotic stability (discrete time)	157
2.29	Theorem: Exponential stability (discrete time)	157
2.30	Example: Matched asymptotic expansion analysis of the reaction equilibrium assumption	169
2.31	Example: Oscillatory dynamics of a nonlinear system	176
2.32	Theorem: Poincaré-Bendixson	189
3.1	Example: Gradient (∇) and Laplacian operators in polar (cylindrical) coordinates	259
3.2	Example: The divergence theorem and conservation laws	267
3.3	Example: Steady-state temperature distribution in a circular cylinder	273
3.4	Example: Transient diffusion in a slab	275
3.5	Example: Steady-state diffusion in a square domain	277
3.6	Example: Eigenfunction expansion for an inhomogeneous problem	278
3.7	Example: Steady diffusion in a cylinder: eigenfunction expansion and multiple solution approaches	279
3.8	Example: Transient diffusion from a sphere	283

3.9	Example: Temperature field around a sphere in a linear gradient	284
3.10	Example: Domain perturbation: heat conduction around a near-sphere	286
3.11	Example: Derivation of a Fourier transform formula	293
3.12	Example: Transient diffusion in an unbounded domain: one and multiple dimensions	294
3.13	Example: Steady diffusion from a wall with an imposed concentration profile	296
3.14	Example: Reaction and diffusion in a membrane	309
3.15	Example: Solving the wave equation	314
4.1	Example: Characteristic function of the normal density	356
4.2	Example: The mean and covariance of the multivariate normal	361
4.3	Example: Characteristic function of the multivariate normal	365
4.4	Example: Marginal normal density	366
4.5	Example: Nonlinear transformation	369
4.6	Example: Maximum of two random variables	369
4.7	Example: Independent implies uncorrelated	371
4.8	Example: Does uncorrelated imply independent?	371
4.9	Example: Independent and uncorrelated are equivalent for normals	373
4.10	Definition: Density of a singular normal	376
4.11	Example: Computing a singular density	377
4.12	Theorem: Normal distributions under linear transformation	379
4.13	Example: Sum of 10 uniformly distributed random variables	382
4.14	Theorem: De Moivre-Laplace central limit theorem	383
4.15	Assumption: Lindeberg conditions	386
4.16	Theorem: Lindeberg-Feller central limit theorem	387
4.17	Theorem: Multivariate CLT—IID	387
4.18	Theorem: Multivariate CLT—Lindeberg-Feller	387
4.19	Example: Conditional normal density	390
4.20	Example: More normal conditional densities	391
4.21	Example: The confidence region, bounding box, and marginal box	397
4.22	Theorem: Mean and variance of samples from a normal	408
4.23	Example: Comparing PCR and PLSR	420
4.24	Theorem: Taylor's theorem with bound on remainder	426

5.1	Example: Diffusion on a plane in Cartesian and polar coordinate systems	466
5.2	Example: Average properties from sampling	468
5.3	Example: Transport of many particles suspended in a fluid	473
5.4	Example: Fokker-Planck equations for diffusion on a plane	474
5.5	Algorithm: First reaction method	483
5.6	Algorithm: Gillespie's direct method or SSA	485
5.7	Example: Observability of a chemical reactor	509
5.8	Theorem: Riccati iteration and estimator stability	511
5.9	Definition: Continuity (with probability one)	515